B. SC (Sem-IV)
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TITLE :- Distribution of Intensity due to F.P.I

Let T and R be the fractions of the incident light intensity, which are transmitted and and refracted at each silvered surface, the fraction amplitudes are $\sqrt{T}$ and $\sqrt{R}$ respectively. Then the amplitude of the wave BG transmitted by AF is $\sqrt{T}$ and of C_1T_1 transmitted by BH is $\sqrt{T}.\sqrt{T}=T$ similarly the amplitude of B_1C_1 arising by reflection of BC_1 at C_1 is $\sqrt{TR}$ and that of B_1C_2 is $R\sqrt{T}$, so that the amplitude of C_2T_2 is C_1T_1, C_2T_2, C_3T_3 ------ are T, TR, TR^2, TR^3 ---- where the amplitude of intial wave is unity. The amplitude and hence ~~where~~ the intensities of transmitted waves decrease systematically with the number of reflections. Neglecting the presence of any sudden phase change in a wave due to reflection at the silvered surfaces, the phase difference ϕ between any two consecutive transmitted beams due to ~~reflection at the silvered surface~~, the phase diff. ~~ϕ between~~ $2d\cos\theta$ in passing to P_2 is

$$\phi = \left(\frac{2\pi}{\lambda}\right) 2d\cos\theta$$

The phase changes by increment of ϕ for each successive transmitted wave. Therefore, if the incident wave be represented by

$$y = \sin\omega t$$

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The light vibration excited by the transmitted waves at the points P_2 of the interference will be transmitted represented by

$$y_1 = T\sin\omega t \qquad y_2 = TR\sin(\omega t - \phi)$$

$$y_3 = TR^2\sin(\omega t - 2\phi) \qquad y_4 = TR^3\sin(\omega t - \phi)$$

and so on. Here we have to consider the interference of practically infinite number of transmitted waves because of small angles of incidence and relatively large area of the fabry-Perot plates. The final result and vibration at P_2 can be written as

$$A\sin(\omega t - \beta)$$

Where A is instantaneous resultant amplitude and β the resultant phase and both are to be determined. By the principle of superposition of waves. the sum of partial vibrations will yield the resulting vibration

$$A\sin(\omega t - \beta) = T\sin\omega t + TR\sin(\omega t - \phi) + TR^2\sin(\omega t - 2\phi) + \cdots \text{ to } \infty$$

or $A[\sin\omega t \cdot \cos\beta - \cos\omega t \cdot \sin\beta] = T[\sin\omega t] + TR(\sin\omega t \cos\phi - \cos\omega t \sin\phi)$
$+ TR^2[\sin\omega t \cos 2\phi - \cos\omega t \sin 2\phi] + \cdots \infty$

Equating the coefficient of $\sin\omega t$ and $\cos\omega t$ on both sides, we have

$$A\cos\beta = T + TR\cos\phi + TR^2\cos 2\phi + TR^3\cos 3\phi + \cdots$$

and $A\sin\beta = TR\sin\phi + TR^2\sin 2\phi + TR^3\sin 3\phi + \cdots$

If I be the resultant intensity at the point of interference in the focal plane of lens is given by

$$I = A^2(\cos^2\beta + \sin^2\beta)$$

$$= A^2(\cos\beta + i\sin\beta)(\cos\beta - \sin\beta)$$

$$= A\cos\beta + iA\sin\beta)(A\cos\beta - iA\sin\beta) \quad \text{—— (2)}$$

Now $A\cos\beta + iA\sin\beta = T + TR(\cos\phi + i\sin\phi) + TR^2(\cos 2\phi + i\sin 2\phi)$
$+ TR^3(\cos 3\phi + i\sin 3\phi) + \cdots$

$$= T + TRe^{i\phi} + TR^2e^{2i\phi} + TR^3e^{3i\phi}$$

$$= T(1 + Re^{i\phi} + R^2e^{2i\phi} + R^3e^{3i\phi} + \cdots)$$

$$= \frac{T}{1 - Re^{i\phi}} \quad \text{—— (3)}$$

Similarly $A\cos\beta - iA\sin\beta = \dfrac{T}{1-Re^{-i\phi}}$ ———— (4)

From eqn(2), the intensity

$$I = \frac{T}{1-Re^{i\phi}} \times \frac{T}{1-Re^{-i\phi}} = \frac{T^2}{(1-Re^{i\phi})(1-Re^{-i\phi})}$$

$$= \frac{T^2}{1-Re^{-i\phi}-Re^{i\phi}+R^2} = \frac{T^2}{1+R^2-2R\cos\phi}$$

$[e^{i\phi}+e^{-i\phi} = 2\cos\phi]$

$$= \frac{T^2}{1+R^2-2R+2R-2R\cos\phi}$$

$$= \frac{T^2}{(1+R)^2+4R\sin^2\frac{\phi}{2}}$$ ———— (5)

Case I I will be max^n. when

$\sin^2\frac{\phi}{2} = I$. i.e $\phi = 2\pi, 4\pi, 6\pi$ --- etc.

$$I_{max} = \frac{T^2}{(1+R^2)}$$ ———— (6)

Case II I will be min when

$\sin^2\left(\frac{\phi}{2}\right) = \pi, 3\pi, 5\pi,$ ---- etc

$$I = \left[\frac{T^2}{1-R^2}\right]$$ ———— (7)

———— x ———— x ————
———— x ———— x ————